



Provable and Robust Wavefront Sensing via Self-Reference Interferometry

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TL;DR Self-interference of wavefront with **spatially shifted copies (with co-prime shift)** and the pairwise phase differences integrated along a **connected graph** enable phase retrieval in a **single, non-iterative pass**, with provable robustness guarantees.

Background

Phase matters

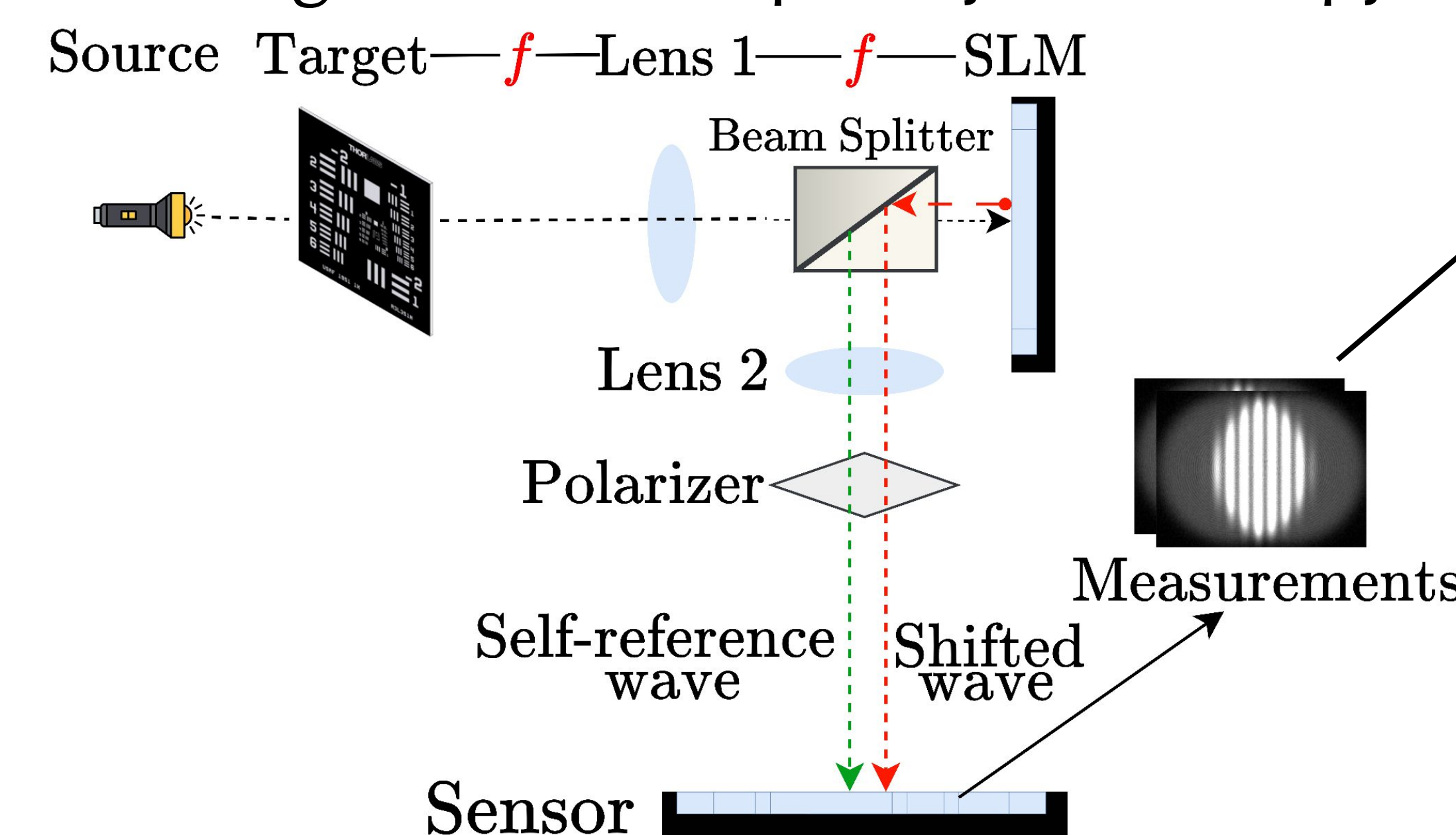
- Light is a wave – with intensity and phase
- Phase carries wavefront information – can enable adaptive optics, aberration correction, and imaging through scattering media.

Phase is missing

- Image sensors record only **intensity**
- Phase retrieval** from intensity is a highly non-convex and (slow) iterative process
- Our goal** – provide an **analytical, non-iterative, provably robust** solution for phase retrieval.

Approach

1. Self-reference interferometry. Interfere incoming wave with a spatially shifted copy



2. Forward Model. Spatial and phase shifts

$$y_{k,q} = \left| \mathbf{x} + e^{j\varphi_q} \mathcal{S}_{\Delta_k}(\mathbf{x}) \right|^2 + \eta_{k,q}$$

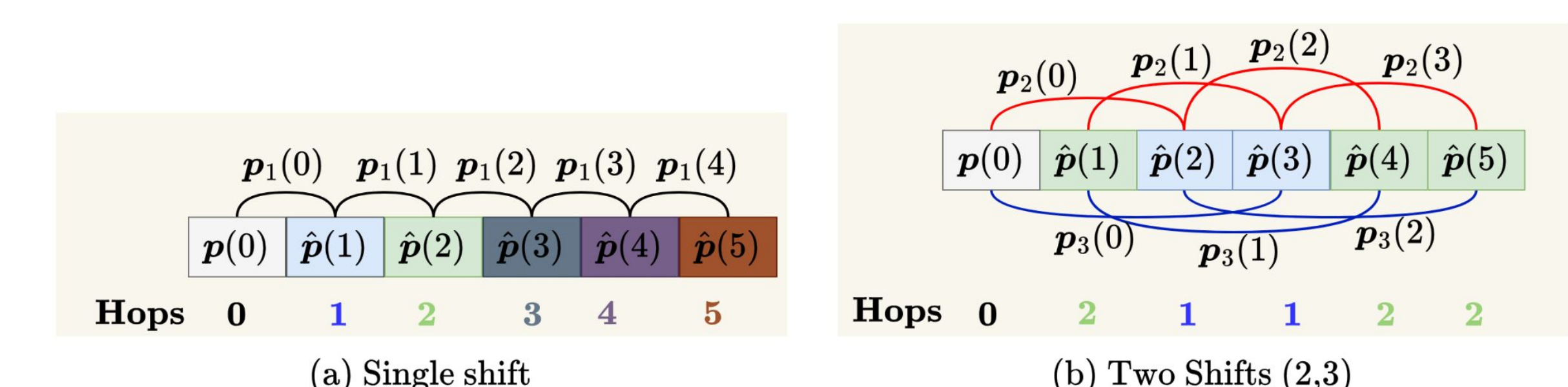
Phase shifts Spatial shifts
 $\varphi_q \in \{0, \pi/2, \pi, 3\pi/2\}$

- Phasor differences can be obtained as

$$\mathbf{p}_k = \frac{\sum_q y_{k,q} e^{-j\varphi_q}}{\left| \sum_q y_{k,q} e^{-j\varphi_q} \right|} \approx e^{j(\mathcal{S}_{\Delta_k}(\phi) - \phi)}$$

- $\mathbf{p}_k(i)$ encodes phase difference between pixel i and $i + \Delta_k$

3. Recovery on a Graph.

- Pixels $\Rightarrow$ nodes; Phasor differences $\Rightarrow$ edges.
 - Fix a reference $\mathbf{p}(0) = 1$ and estimate $\hat{\mathbf{p}}(i + \Delta_k) = \hat{\mathbf{p}}(i) \mathbf{p}_k(i)$
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- Error grows with **propagation distance** from the reference.
 - Single shift:** long propagation paths
 - Two shifts:** fewer hops and less noise accumulation.

Challenges

Common approaches have the following limitations

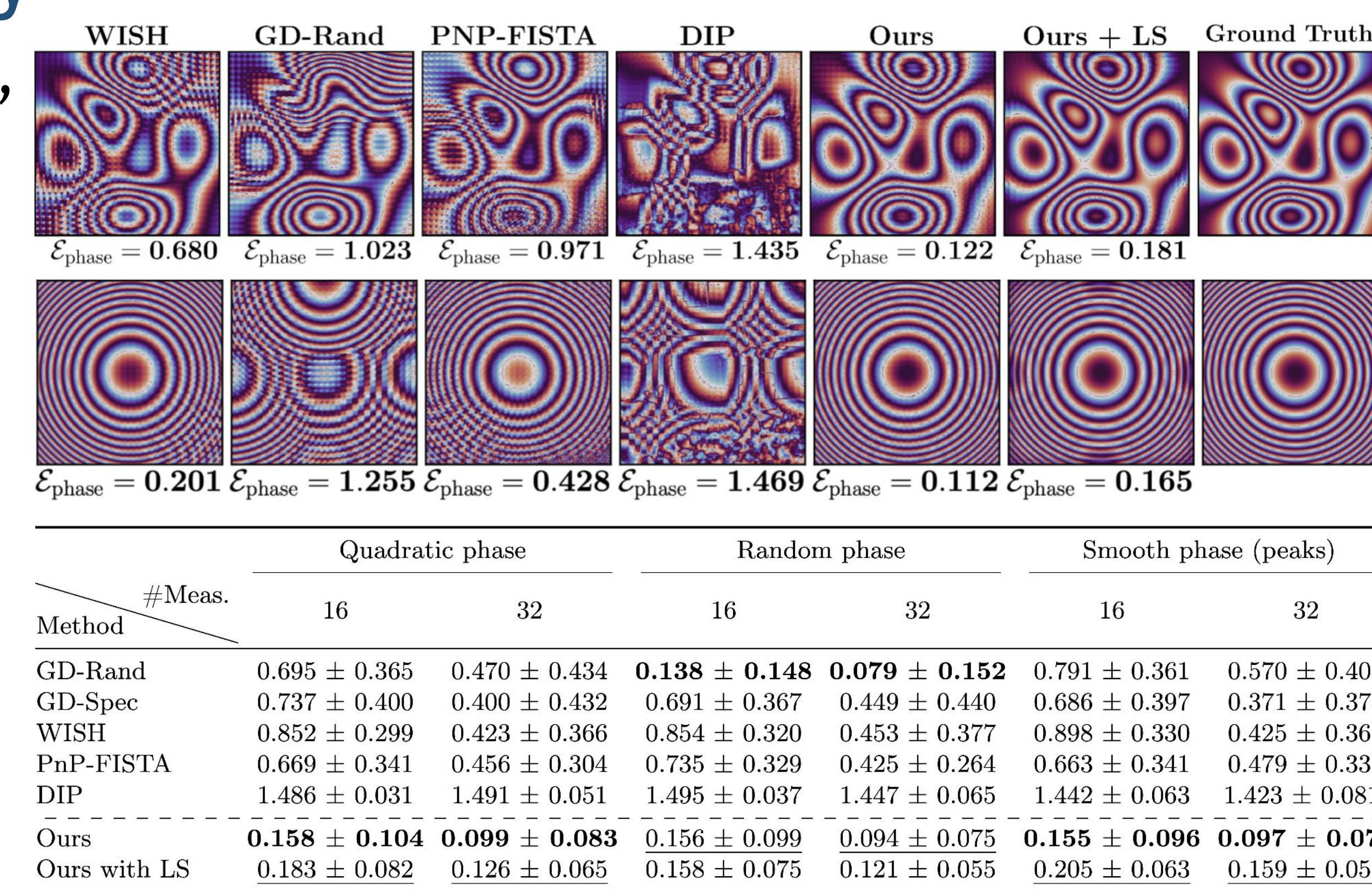
- Phase-shifting interferometry (PSI):** needs a stable external reference beam, which can be bulky and vibration-sensitive
- Self-reference / Point-diffraction:** The self-reference is very dim $\Rightarrow$ low SNR
- Slope-based sensors (Shack–Hartmann, pyramid):** limited resolution, noise-sensitive
- Iterative phase retrieval (Gerchberg-Saxton, deep priors):** non-convex, slow, limited guarantees on convergence and accuracy

Simulated Experiments: Phase Recovery

Simulated scenes, quadratic and smooth phase, realistic sensor noise, compared against multiple baselines (e.g., 16 or 32 measurements with 2 or 4 spatial shifts in horizontal/vertical directions and 4 phase shifts).

Our method

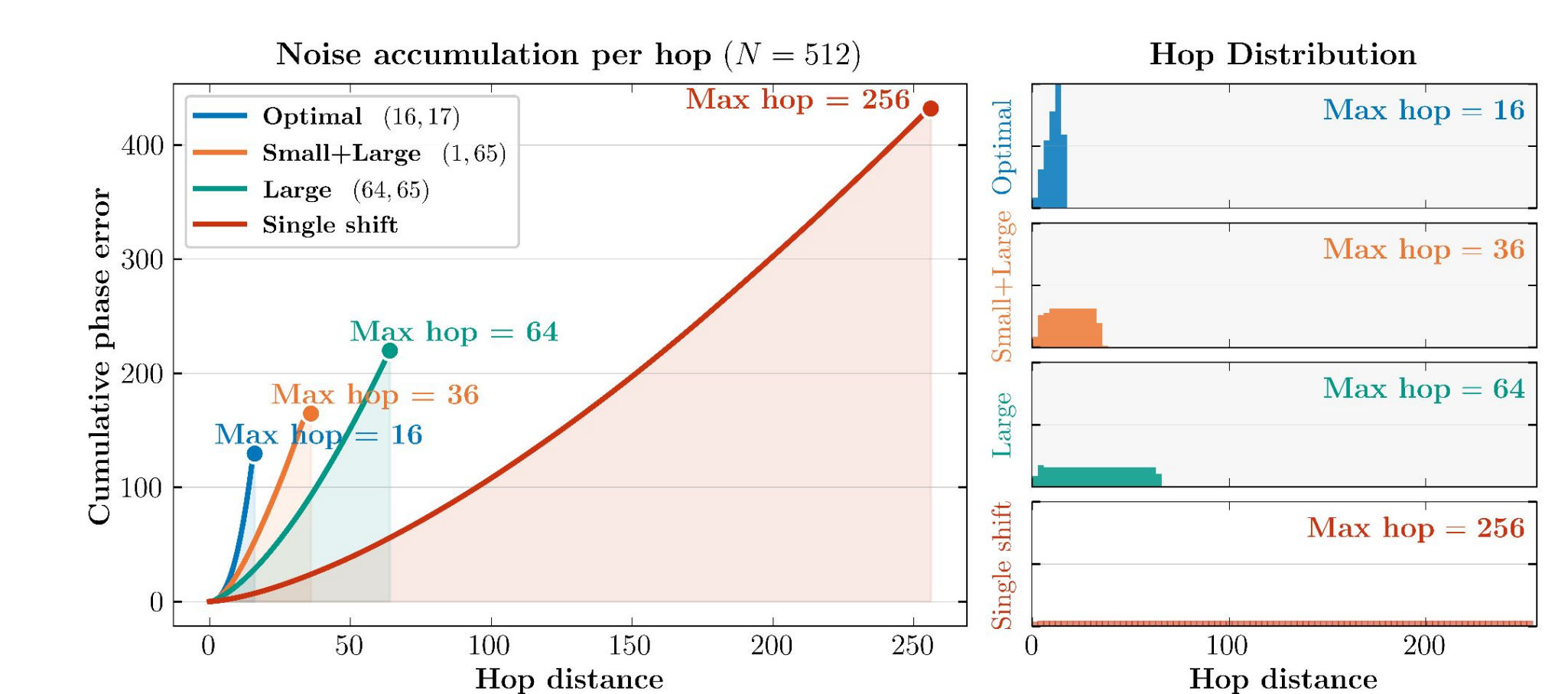
- achieves best or second-best accuracy across phase type and measurement count.
- recovers phase from as few as 8 measurements and runs 8 $\times$ faster.



Optimal Shifts and Robustness

Question. Which shift pair (s, t) should we pick?

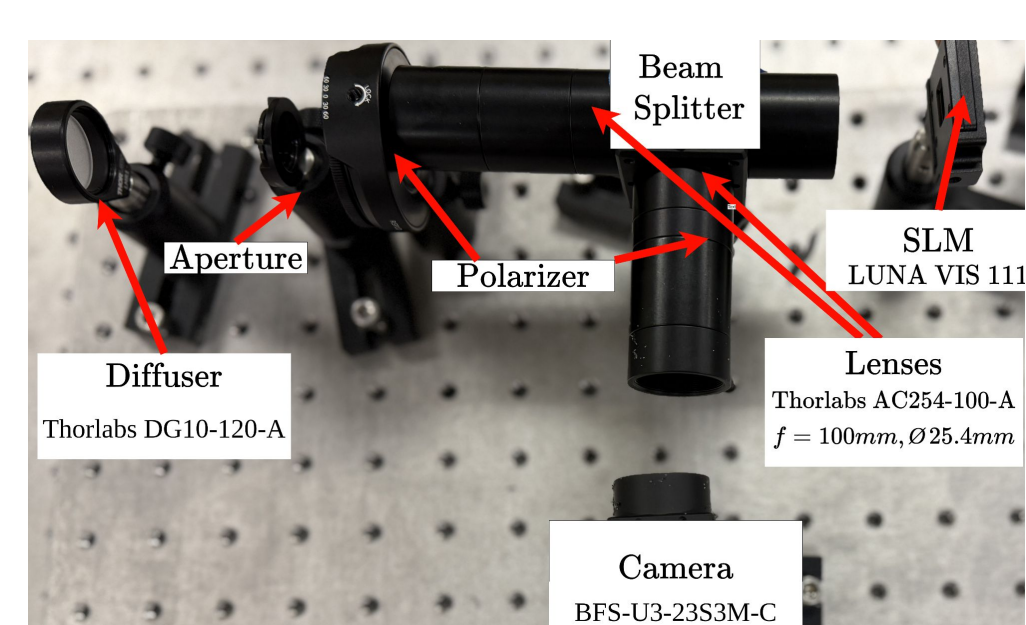
- Connectivity** is guaranteed with **co-prime shifts**
- Optimal pair** $s = \lfloor \sqrt{\frac{N}{2}} \rfloor, t = s + 1$ **minimizes the maximum hop** and bounds worst-case error.



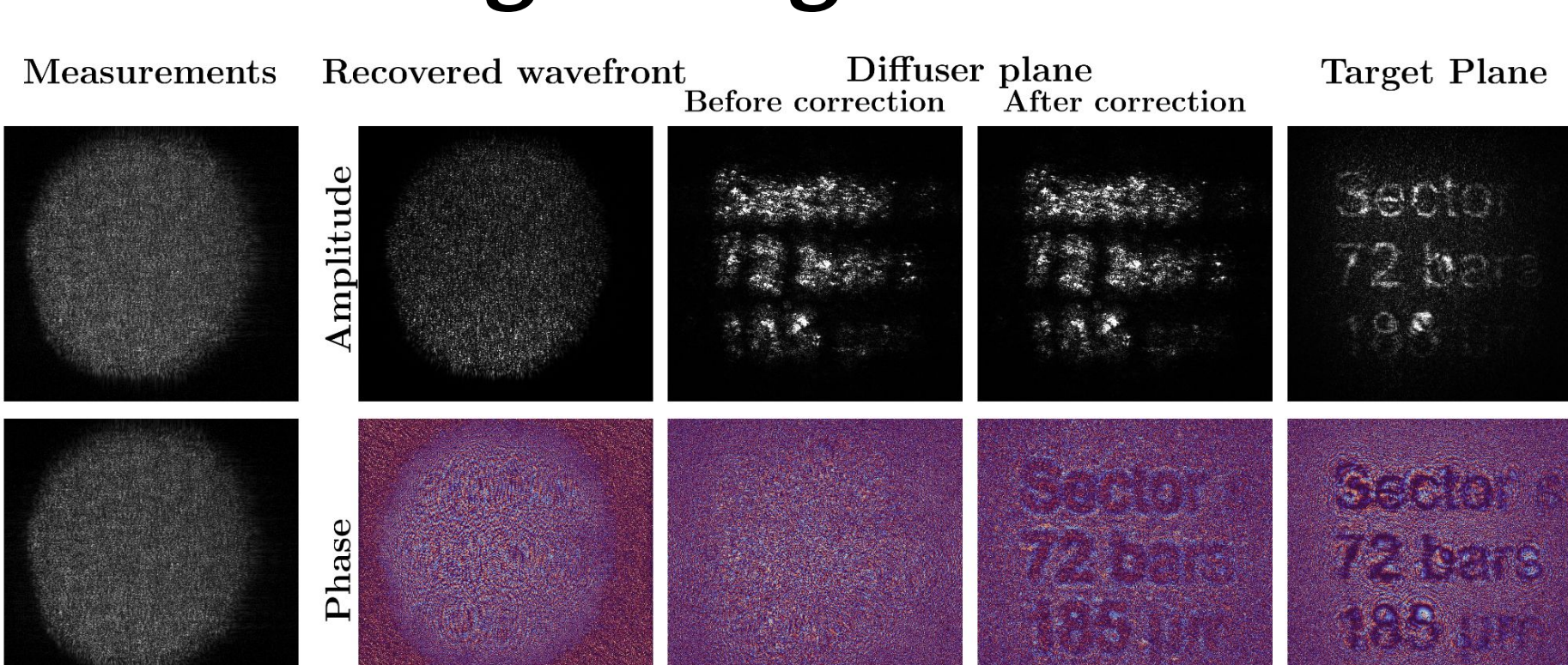
Verification of worst-case error accumulation (1D signal)

Real Hardware Experiments

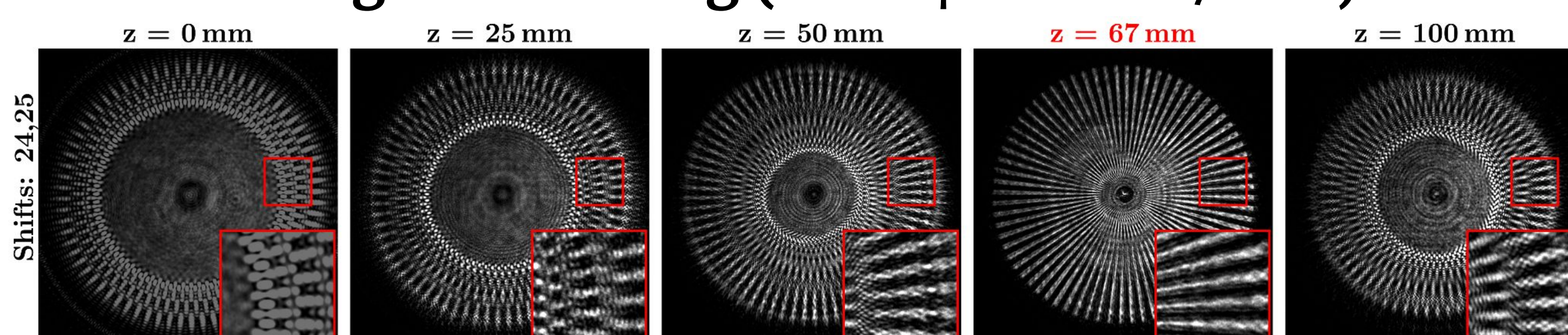
Hardware prototype



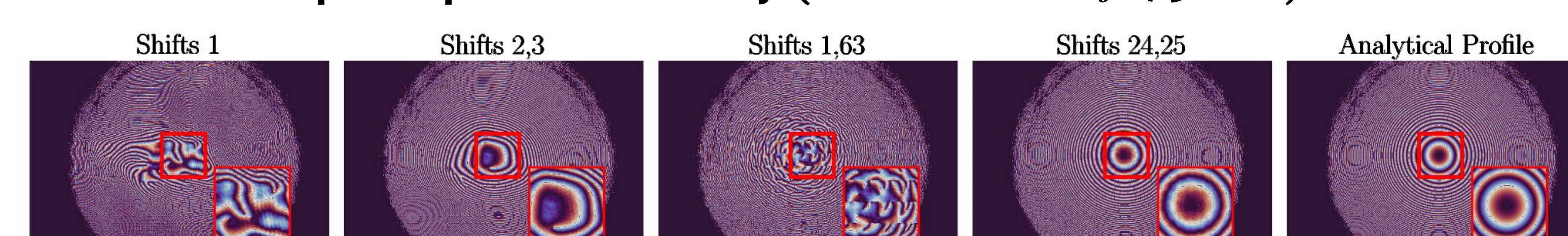
Seeing through a diffuser



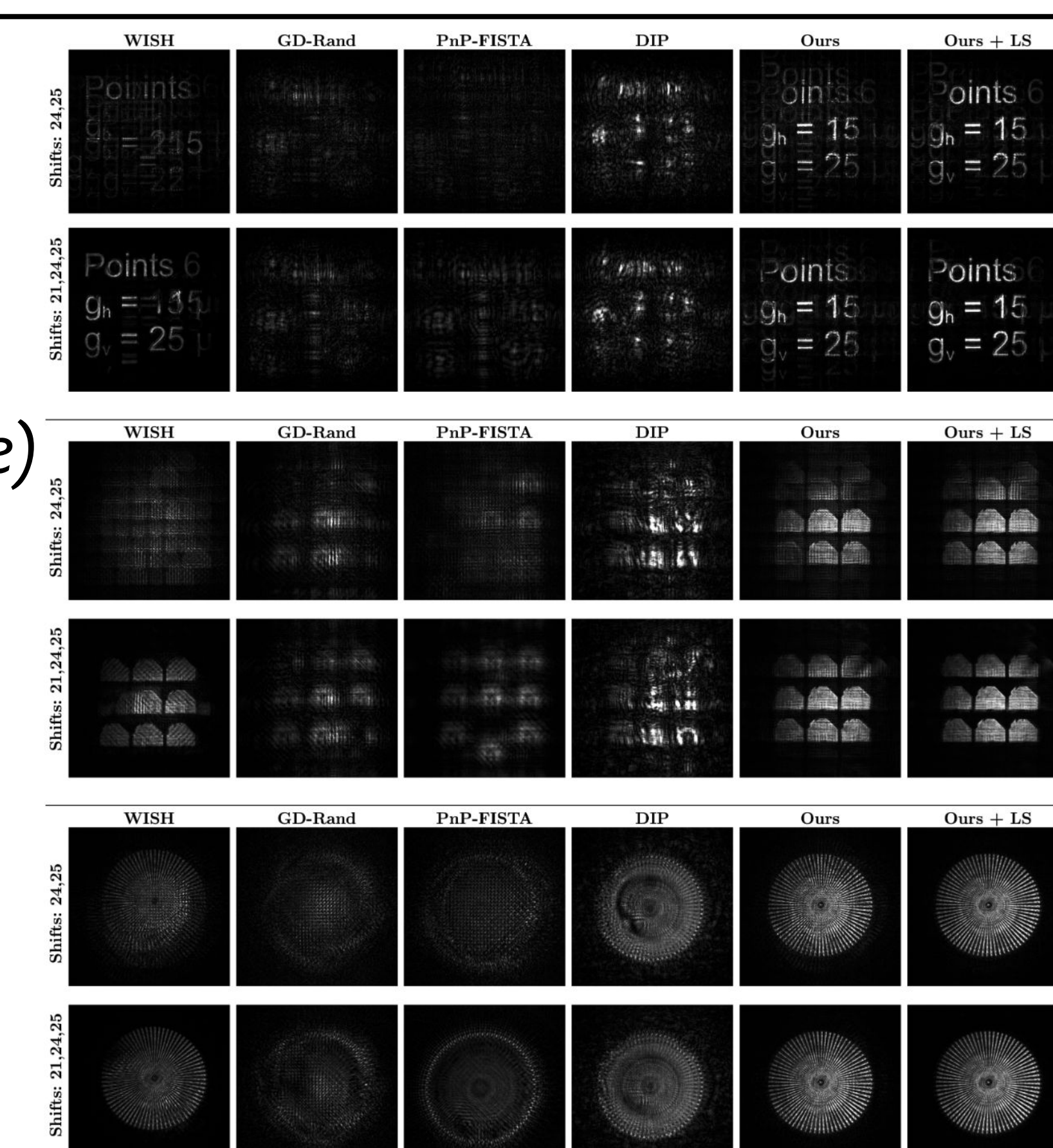
Target refocusing (focus point at 67 mm)



Optical profile recovery (biconvex lens $f=75$ mm)



Qualitative comparison with existing methods (our method is faster and accurate)



Conclusion

- A **simple, robust** method for wavefront sensing without an external reference beam.
- Analytical, non-iterative** phase recovery via graph propagation **with provable guarantees**.
- Further improvements possible by **averaging multiple paths** and least-squares refinement.
- Validated through simulations and real hardware experiments.

Acknowledgements. This work is partially supported by NSF CCF-2046293 and CCF-2504614.
More info: <https://csiplab.github.io/coprime-psi/>